The New Normal cannot be charted with traditional stress tests – How can science help?

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1. First Generation Stress Tests
2. Systematic Stress Tests for the Trading Book
3. New Developments

Outline

1. First Generation Stress Tests
2. Systematic Stress Tests for the Trading Book
3. New Developments

Purpose of Stress Testing: Complement statistical risk measurement

- Stress Tests: Which scenarios lead to big losses?
  Derive risk reducing action.
  (Statistical risk measurements: What are prob's of big losses?)
- Stress Tests: Address model risk.
  Consider alternative risk factor distribution.
  (Statistical risk measurement: Assume fixed model.)

Requirements on stress scenarios (Basel II)

- plausible
- severe
- suggestive of risk reducing action

See Basel Principles of Sound Stress Testing

Framework

- Trading book: "held for trading"
  Portfolio value function $X(r)$
  on risk factor space $\Omega \subset \mathbb{R}^n$.
  Scenarios: Alternative realisations of risk factor vector $\boldsymbol{r} \in \Omega$.
  portfoliofunction.pdf
- Banking book: "held at fair value"
  Actuarial valuation of portfolio: $\mathbb{E}_{\mathbb{P}_0}(X)$
  with reference risk factor distribution $\mathbb{P}_0$
  on risk factor space $\Omega \subset \mathbb{R}^n$.
  Scenarios: Alternative risk factor distributions Q on Ω.

First Generation Stress Tests: Hand-picked Point Scenarios

- Point scenario: each risk factor gets a value: $\boldsymbol{r} \in \Omega$
- A small number of scenarios is picked by hand, ideally involving heterogeneous groups of experts.

$$A = \{\boldsymbol{r}^1, \boldsymbol{r}^2, \dots, \} \subset \Omega$$

  a small set of hand-picked scenarios.
- Find worst case scenario and worst case loss in A

$$\min_{\boldsymbol{r} \in A} X(\boldsymbol{r})$$

- Worst case loss over A is a coherent risk measure.

First Generation Stress Tests: Examples

- most stress tests of market or credit risk performed by financial institutions
- SPAN rules
- FSAP stress tests
- US institutional stress tests during 2009 crisis
- 2014 stress tests of ECB
- All recent stress tests of IMF, EBA, national authorities

Criticism of First Generation Stress Tests

Accidental or deliberate misrepresentation of risks:

1. Neglecting severe but plausible scenarios
   $\rightarrow$ possible illusion of safety
2. Considering too implausible scenarios
   $\rightarrow$ possible premature reaction to stress test results

Outline

Second Generation Stress Tests: Plausible Scenarios

- Measure of plausibility for point scenarios:

$$\mathrm{Maha}(\boldsymbol{r}) := \sqrt{(\boldsymbol{r} - \mathbb{E}(\boldsymbol{r}))^T \cdot \Sigma^{-1} \cdot (\boldsymbol{r} - \mathbb{E}(\boldsymbol{r}))},$$

  where Σ is covariance matrix of risk factor distribution $\mathbb{P}_0$.
- Intuition:
  Scenarios in which some risk factors move **many standard deviations** are implausible.
  Scenarios in which some pair of risk factors moves **against their correlation** are implausible.
- Note: The definition of plausibility requires the specification of a risk factor distribution ($\mathbb{P}_0$).

Second Generation Stress Tests: Systematic Point Scenario Analysis

- Set of plausible scenarios

$$A := \mathrm{Ell}_h := \{\boldsymbol{r} : \mathrm{Maha}(\boldsymbol{r}) \leq h\},$$

where h is the plausibility threshold.

- Systematic search of worst case scenario:

$$\min_{\boldsymbol{r} \in \mathrm{Ell}_h} X(\boldsymbol{r}) \tag{1}$$

- Note: The probability mass of Ell_h depends not just on h, but also on the number of dimensions. For example, if risk factors are normally distributed, the probability mass of Ell_h is

$$\frac{2^{-n/2}}{\Gamma(\frac{n}{2})} \int_0^h t^{n/2-1} \exp(-\frac{t}{2}) dt.$$

Second generation stress of linear portfolio

Portfolio value is linear function of normal risk factors:

$$X(\boldsymbol{r}) = \boldsymbol{x}^T(\boldsymbol{\mu} - \boldsymbol{r})$$

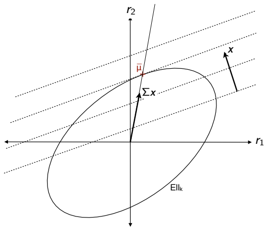
$$\mathbb{P}_0 \sim N(\boldsymbol{\mu}, \Sigma).$$

Systematic search of worst case scenario eq. (1):

$$\min_{\boldsymbol{r} \in \mathrm{Ell}_h} \boldsymbol{x}^T(\boldsymbol{\mu} - \boldsymbol{r}).$$

Solution of (1):

- Worst case scenario: $\overline{\boldsymbol{\mu}} = \boldsymbol{\mu} - \frac{h}{\sqrt{\boldsymbol{x}^T \Sigma \boldsymbol{x}}} \Sigma \boldsymbol{x}$
- Worst case loss: $h\sqrt{\boldsymbol{x}^T \Sigma \boldsymbol{x}}$.



Second generation stress of general portfolio

- Analytic solutions for worst case problem are available if portfolio function X is linear or quadratic.
- For other portfolio value functions, numerical worst case search algorithms are available.

Advantages of Systematic Stress Testing with Point Scenarios

All three requirements on stress testing are met:

- Do not miss plausible but severe scenarios.
- Do not consider scenarios which are too implausible.
- Worst case scenario over Ell_h gives information about portfolio structure and suggests risk reducing action.

Identifying risk reducing action from worst case scenario

Define the loss contribution of risk factor i in the worst case scenario r as

$$\frac{X(\mu) - X(\mu_1, \ldots, \mu_{i-1}, r_i, \mu_{i+1}, \ldots \mu_n)}{X(\mu) - X(r)}, \tag{2}$$

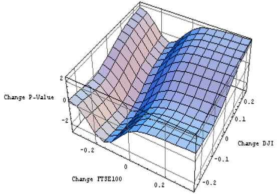
where μ is the present scenario.

The risk factors with the highest loss contribution in the worst case scenario are the key risk factors to be hedged first.

Design rule for hedge: Take hedge positions with sufficiently high payoff if key risk factor takes its worst case value.

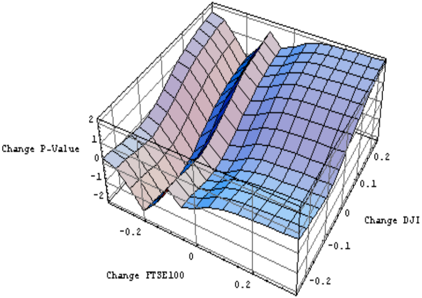
Advantages of Systematic Stress Testing with Point Scenarios

	Risk Factors	Rel Changes	Rel. Loss	Loss Contribution
Report 1	FTSE100	-13%	206%	74%
Report 2	FTSE100 DJI	-13% -8%	264%	94%
Report 3	FTSE100 DJI NIK225	-13% -8% -5%	271%	97%



Hedge against Worst Case Stress

	Original portfolio	Hedged portfolio
worst rel. loss	-279%	-115%
worst abs. loss	3.35m	1.26m
insurance cost	-	0.04m



Problems of Systematic Stress Testing with Point Scenarios

1. What if risk factor distributions $\mathbb{P}_0$ is non-elliptical?
2. What if risk true factor distribution is not $\mathbb{P}_0$?
   Model risk is not addressed.

3. Maha does not take into account fatness of tails.
4. $\mathrm{MaxLoss}$ over Ell_h depends on choice of coordinates.

Outline

Worst Case Scenarios for Banking Book

- **Set of plausible scenarios:** Instead of ellipsoid take Kullback-Leibler sphere in the space of distributions

$$A := \Gamma(\mathbb{P}_0, k) := \{Q : I(Q||\mathbb{P}_0) \leq k\}.$$

- **Severity of banking book scenarios:** $\mathbb{E}_Q(X)$
- Systematic stress test for the banking book:

$$\inf_{Q \in \Gamma(\mathbb{P}_0, k)} \mathbb{E}_Q(X) \tag{3}$$

If it exists, call scenario achieving MaxLoss: $\overline{Q}$.

The Basic Tool

- Tool from large deviations theory for solving explicitly the optimisation problem (3):

$$G(\theta_2) := \log \left(\int e^{\theta_2 X(\boldsymbol{r})} d\mathbb{P}_0(\boldsymbol{r}) \right),$$

for $\theta_2 < 0$.

Thermodynamic counterpart of G: log of partition function Z.
Thermodynamic counterpart of $-\theta_2$: absolute temperature β.

Solution of Worst Case: The Generic Case

Theorem

- *Except in the pathological cases, the equation*

$$\theta_2 G'(\theta_2) - G(\theta_2) = k, \tag{4}$$

has always a unique negative solution. Call it $\overline{\theta}_2$*.*

- *The worst alternative distribution* $\overline{Q}$ *is the distribution with* $\mathbb{P}_0$*-density*

$$\frac{d\overline{Q}}{d\mathbb{P}_0}(\boldsymbol{r}) = e^{\overline{\theta}_2 X(\boldsymbol{r}) - G(\overline{\theta}_2)}, \tag{5}$$

- *The Maximum Loss achieved in the mixed worst case scenario* $\overline{Q}$ *is*

$$\mathbb{E}_{\overline{Q}}(X) = G'(\overline{\theta}_2).$$

Example: Stressed transition probabilities

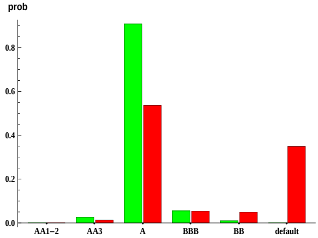
- $\Omega = \{0, 1, \ldots n\}$: rating classes.
- $\mathbb{P}_0$: $\boldsymbol{p} = (p_1, \ldots, p_n)$: estimated transition probabilities
- $\boldsymbol{x} = (x_1, \ldots, x_n)$: profits after transitions
- $G(\theta_2) = \log\left(\sum_{j=1}^{n} p_j \exp(\theta_2 x_j)\right)$.
- Worst case transition probabilities: $\overline{p}_i = \frac{p_i \exp(\overline{\theta}_2 x_i)}{\sum_{j=1}^{n} p_j \exp(\overline{\theta}_2 x_j)}$.

Stressed transition probabilites
Numerical example: A-rated bond

	AA1-2	AA3	A	BBB	BB	Default
profit from transitions [%]	3.20%	1.07%	0.00%	-3.75%	-15.83%	-51.80%
est'd trans. prob. [%]	0.09	2.60	90.75	5.50	1.00	0.06
worst c. trans. prob. [%]	0.036	1.34	53.53	5.37	4.91	34.8

present value of the bond:
Expected payoff change from transitions under est'd probs: -0.37%

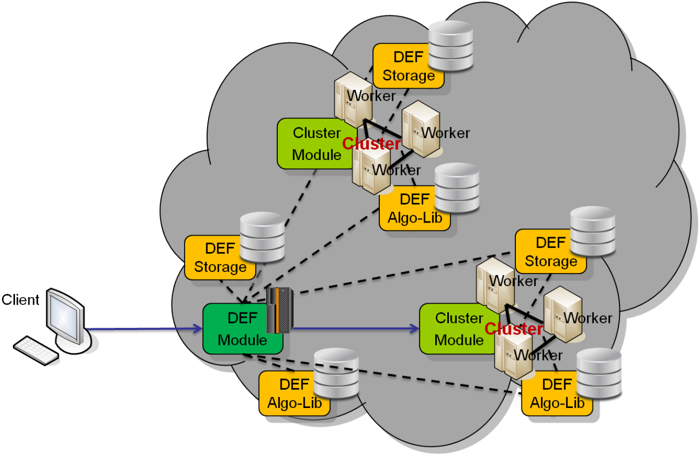
worst case value of the bond:
Expected payoff change from transitions under worst case probs at k=2: -19.07%



Numerical Feasibility: How long do systematic stress tests take?

- It depends on the number of variables, and on the accuracy required.
  Sometimes tens of thousands of scenarios have to be evaluated.
- Distributed Execution Framework:
  Scalable use of arbitrary hardware infrastructure, in house or in the cloud. If desired with thousands of workers.

Numerical Feasibility:
How long do systematic stress tests take?



Open question: Systemic effects in stress tests

- How do banks' reactions to stress event influence each other?
- New price drops caused by fire sales of others.
- New counterparty defaults caused by stress.

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