The New Normal cannot be charted with traditional stress tests – How can science help?

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BVI-Risikomanagementtag
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1. First Generation Stress Tests

2. Systematic Stress Tests for the Trading Book

3. New Developments
1 First Generation Stress Tests

2 Systematic Stress Tests for the Trading Book

3 New Developments
Purpose of Stress Testing:
Complement statistical risk measurement

- Stress Tests: Which scenarios lead to big losses?
  Derive risk reducing action.
  (Statistical risk measurements: What are prob’s of big losses?)
- Stress Tests: Address model risk.
  Consider alternative risk factor distribution.
  (Statistical risk measurement: Assume fixed model.)
Requirements on stress scenarios (Basel II)

- plausible
- severe
- suggestive of risk reducing action

See Basel Principles of Sound Stress Testing
Framework

- **Trading book:** “held for trading”
  Portfolio value function $X(r)$ on risk factor space $\Omega \subset \mathbb{R}^n$.
  Scenarios: Alternative realisations of risk factor vector $r \in \Omega$.

- **Banking book:** “held at fair value”
  Actuarial valuation of portfolio: $\mathbb{E}_{P_0}(X)$ with reference risk factor distribution $P_0$
  on risk factor space $\Omega \subset \mathbb{R}^n$.
  Scenarios: Alternative risk factor distributions $Q$ on $\Omega$. 
First Generation Stress Tests: Hand-picked Point Scenarios

- **Point scenario:** each risk factor gets a value: \( r \in \Omega \)
- A small number of scenarios is picked by hand, ideally involving heterogeneous groups of experts.

\[ A = \{ r^1, r^2, \ldots, \} \subset \Omega \]

a small set of hand-picked scenarios.

- Find worst case scenario and worst case loss in \( A \)

\[
\min_{r \in A} X(r)
\]

- Worst case loss over \( A \) is a coherent risk measure.
First Generation Stress Tests:  
Examples

• most stress tests of market or credit risk performed by financial institutions
• SPAN rules
• FSAP stress tests
• US institutional stress tests during 2009 crisis
• 2014 stress tests of ECB
• All recent stress tests of IMF, EBA, national authorities
Criticism of First Generation Stress Tests

Accidental or deliberate misrepresentation of risks:

1. Neglecting severe but plausible scenarios → possible illusion of safety
2. Considering too implausible scenarios → possible premature reaction to stress test results
Outline

1 First Generation Stress Tests

2 Systematic Stress Tests for the Trading Book

3 New Developments
Second Generation Stress Tests: Plausible Scenarios

• Measure of plausibility for point scenarios:

\[
\text{Maha}(r) := \sqrt{(r - \mathbb{E}(r))^T \cdot \Sigma^{-1} \cdot (r - \mathbb{E}(r))},
\]

where \(\Sigma\) is covariance matrix of risk factor distribution \(\mathbb{P}_0\).

• Intuition:
Scenarios in which some risk factors move many standard deviations are implausible.
Scenarios in which some pair of risk factors moves against their correlation are implausible.

• Note: The definition of plausibility requires the specification of a risk factor distribution \((\mathbb{P}_0)\).
Second Generation Stress Tests: Systematic Point Scenario Analysis

• Set of plausible scenarios

\[ A := \operatorname{Ell}_h := \{ r : \text{Maha}(r) \leq h \} , \]

where \( h \) is the plausibility threshold.

• Systematic search of worst case scenario:

\[ \min_{r \in \operatorname{Ell}_h} X(r) \] (1)

• Note: The probability mass of \( \operatorname{Ell}_h \) depends not just on \( h \), but also on the number of dimensions. For example, if risk factors are normally distributed, the probability mass of \( \operatorname{Ell}_h \) is

\[ \frac{2^{-n/2}}{\Gamma(n/2)} \int_0^h t^{n/2-1} \exp\left(-\frac{t}{2}\right) dt. \]
Second generation stress of linear portfolio

Portfolio value is linear function of normal risk factors:

\[ X(r) = x^T (\mu - r) \]

\[ \mathbb{P}_0 \sim N(\mu, \Sigma). \]

Systematic search of worst case scenario eq. (1):

\[ \min_{r \in \text{Ell}_h} x^T (\mu - r). \]

Solution of (1):

- Worst case scenario: \( \bar{\mu} = \mu - \frac{h}{\sqrt{x^T \Sigma x}} \Sigma x \)
- Worst case loss: \( h \sqrt{x^T \Sigma x} \).
Second generation stress of general portfolio

- Analytic solutions for worst case problem are available if portfolio function $X$ is linear or quadratic.
- For other portfolio value functions, numerical worst case search algorithms are available.
Advantages of Systematic Stress Testing with Point Scenarios

All three requirements on stress testing are met:

• Do not miss plausible but severe scenarios.
• Do not consider scenarios which are too implausible.
• Worst case scenario over $\mathbb{E}l_h$ gives information about portfolio structure and suggests risk reducing action.
Identifying risk reducing action from worst case scenario

Define the loss contribution of risk factor $i$ in the worst case scenario $r$ as

$$X(\mu) - X(\mu_1, \ldots, \mu_{i-1}, r_i, \mu_{i+1}, \ldots \mu_n) \over X(\mu) - X(r),$$

where $\mu$ is the present scenario.

The risk factors with the highest loss contribution in the worst case scenario are the key risk factors to be hedged first.

Design rule for hedge: Take hedge positions with sufficiently high payoff if key risk factor takes its worst case value.
Advantages of Systematic Stress Testing with Point Scenarios

<table>
<thead>
<tr>
<th>Risk Factors</th>
<th>Rel Changes</th>
<th>Rel. Loss</th>
<th>Loss Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Report 1 FTSE100</td>
<td>-13%</td>
<td>206%</td>
<td>74%</td>
</tr>
<tr>
<td>Report 2 FTSE100 DJI</td>
<td>-13% -8%</td>
<td>264%</td>
<td>94%</td>
</tr>
<tr>
<td>Report 3 FTSE100 DJI NIK225</td>
<td>-13% -8% -5%</td>
<td>271%</td>
<td>97%</td>
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</table>
Hedge against Worst Case Stress

<table>
<thead>
<tr>
<th></th>
<th>Original portfolio</th>
<th>Hedged portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>worst rel. loss</td>
<td>-279%</td>
<td>-115%</td>
</tr>
<tr>
<td>worst abs. loss</td>
<td>3.35m</td>
<td>1.26m</td>
</tr>
<tr>
<td>insurance cost</td>
<td>-0.04m</td>
<td>0.04m</td>
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</tbody>
</table>
Problems of Systematic Stress Testing with Point Scenarios

1. What if risk factor distributions $\mathbb{P}_0$ is non-elliptical?
2. What if risk true factor distribution is not $\mathbb{P}_0$?
   Model risk is not addressed.

3. Maha does not take into account fatness of tails.
4. MaxLoss over $\text{Ell}_h$ depends on choice of coordinates.
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Worst Case Scenarios for Banking Book

- **Set of plausible scenarios:** Instead of ellipsoid take Kullback-Leibler sphere in the space of distributions

  \[ A := \Gamma(P_0, k) := \{ Q : I(Q||P_0) \leq k \} \, . \]

- **Severity of banking book scenarios:** \( \mathbb{E}_Q(X) \)

- Systematic stress test for the banking book:

  \[ \inf_{Q \in \Gamma(P_0, k)} \mathbb{E}_Q(X) \tag{3} \]

  If it exists, call scenario achieving MaxLoss: \( \overline{Q} \).
The Basic Tool

- Tool from large deviations theory for solving explicitly the optimisation problem (3):

\[ G(\theta_2) := \log \left( \int e^{\theta_2 X(r)} d\mathbb{P}_0(r) \right), \]

for \( \theta_2 < 0 \).

Thermodynamic counterpart of \( G \): log of partition function \( Z \).
Thermodynamic counterpart of \( -\theta_2 \): absolute temperature \( \beta \).
Solution of Worst Case: The Generic Case

Theorem

- **Except in the pathological cases, the equation**

\[ \theta_2 G'(\theta_2) - G(\theta_2) = k, \]  

*(4)*

*has always a unique negative solution. Call it \( \bar{\theta}_2 \).*

- **The worst alternative distribution \( \overline{Q} \) is the distribution with \( \mathbb{P}_0 \)-density**

\[ \frac{d \overline{Q}}{d \mathbb{P}_0}(r) = e^{\bar{\theta}_2 X(r) - G(\bar{\theta}_2)}, \]  

*(5)*

- **The Maximum Loss achieved in the mixed worst case scenario \( \overline{Q} \) is**

\[ \mathbb{E}_{\overline{Q}}(X) = G'(\bar{\theta}_2). \]
Example: Stressed transition probabilities

- $\Omega = \{0, 1, \ldots, n\}$: rating classes.
- $\mathbb{P}_0: \mathbf{p} = (p_1, \ldots, p_n)$: estimated transition probabilities
- $\mathbf{x} = (x_1, \ldots, x_n)$: profits after transitions
- $G(\theta_2) = \log \left( \sum_{j=1}^{n} p_j \exp(\theta_2 x_j) \right)$.
- Worst case transition probabilities: $\bar{p}_i = \frac{p_i \exp(\bar{\theta}_2 x_i)}{\sum_{j=1}^{n} p_j \exp(\bar{\theta}_2 x_j)}$. 
Stressed transition probabilities
Numerical example: A-rated bond

<table>
<thead>
<tr>
<th></th>
<th>AA1-2</th>
<th>AA3</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>Default</th>
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</thead>
<tbody>
<tr>
<td>profit from transitions [%]</td>
<td>3.20%</td>
<td>1.07%</td>
<td>0.00%</td>
<td>-3.75%</td>
<td>-15.83%</td>
<td>-51.80%</td>
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<tr>
<td>est’d trans. prob. [%]</td>
<td>0.09</td>
<td>2.60</td>
<td>90.75</td>
<td>5.50</td>
<td>1.00</td>
<td>0.06</td>
</tr>
<tr>
<td>worst c. trans. prob. [%]</td>
<td>0.036</td>
<td>1.34</td>
<td>53.53</td>
<td>5.37</td>
<td>4.91</td>
<td>34.8</td>
</tr>
</tbody>
</table>

present value of the bond:
Expected payoff change from transitions under est’d probs: -0.37%

worst case value of the bond:
Expected payoff change from transitions under worst case probs at k=2: -19.07%
Numerical Feasibility:
How long do systematic stress tests take?

- It depends on the number of variables, and on the accuracy required. Sometimes tens of thousands of scenarios have to be evaluated.
- Distributed Execution Framework: Scalable use of arbitrary hardware infrastructure, in house or in the cloud. If desired with thousands of workers.
Numerical Feasibility:

How long do systematic stress tests take?
Open question: Systemic effects in stress tests

- How do banks’ reactions to stress event influence each other?
- New price drops caused by fire sales of others.
- New counterparty defaults caused by stress.
Some references


• Breuer T., M. Jandacka, K. Rheinberger, M. Summer: How to find plausible, severe, and useful stress scenarios, International Joural of Central Banking 5 (2009), 205-224


